

DEVELOPMENT OF ML STRATEGIES FOR ARRHYTHMIA DIAGNOSIS USING MULTIVARIATE CARDIAC ECG SIGNALS

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Abstract:

The paper describes the research work done so far in the field of Machine Learning (ML) technologies for a reliable diagnosis of cardiac arrhythmias with the help of multivariate Electrocardiogram (ECG) signals. The proposed system uses top-notch preprocessing methods like filtering the noise, baseline shifting, and wavelet transformation so as to improve the quality of ECG and the extraction of its features. The morphological, temporal, and statistical features are the ones to train the optimized classifiers, namely support vector machine (SVM) and multilayer perceptron (MLP). The classifier's optimization process incorporates particle swarm optimization (PSO) which facilitates both the convergence and accuracy of the classifiers. The models are tested using benchmark ECG datasets with several metrics including accuracy, sensitivity, specificity, and F1-score. The results of the experiments indicate that the hybrid PSO-SVM technique is superior to the traditional methods by providing better precision and robustness in the classification of arrhythmia. This study emphasizes the power of ML-based diagnostic frameworks in the early discovery of cardiac abnormalities and in intelligent healthcare decision support systems.

Keywords: *Arrhythmia Diagnosis, Electrocardiogram, Machine Learning, Multilayer Perceptron, Particle Swarm Optimization, Signal Processing.*

I. INTRODUCTION

Cardiovascular diseases are the prime culprits of death globally and arrhythmia is one of the most frequent and dangerous forms of the disease. Arrhythmia is a term that describes the deviation from normal heartbeat rhythm, which can be traced to the heart's electrical system generating abnormally. Early, accurate, and precise detection of the arrhythmias is very important for the

initiation of timely intervention and thereby effective treatment. Electrocardiogram (ECG) signals that depict the heart's electrical activity, function as a mainstay of diagnostics for these irregularities. But, the manual of ECG signals is time-consuming, and prone to errors, and relies on clinical expertise. As a solution to the said drawbacks, machine learning (ML) techniques are the most prevalent and accepted way for the automatic detection and classification of arrhythmia. The ML algorithms will quickly and accurately perform the analysis of multivariate ECG signals, extraction of complex features, detection of subtle patterns which are characteristic of the anomalies in the heart. The research work concentrates on the creation of optimized ML techniques, mainly Support Vector Machine (SVM) and Multilayer Perceptron (MLP) plus Particle Swarm Optimization (PSO) for reaching a better quality and generalization. The method promises inclusion of top-notch preprocessing, feature extraction using wavelet transforms, and parameter tuning so as to uplift the performance. By putting the models to the test on standard ECG datasets, the study seeks to build a strong and clever diagnostics system that can support medical staff in the process of detecting arrhythmia. The research will eventually result in the development of automated cardiac monitoring systems, which in turn will support precision medicine and improve patient outcomes.

II. LITERATURE SURVEY

Recent advances in ECG-based arrhythmia detection have heavily leveraged machine learning and deep learning techniques to improve accuracy and enable real-time monitoring. Baca and Valdivia (2025) proposed an efficient deep learning framework for arrhythmia detection using

smartwatch ECG recordings, highlighting the potential of wearable devices in capturing high-quality cardiac signals and enabling continuous monitoring in daily life. Similarly, Abdelrazik et al. (2025) focused on wearable devices for arrhythmia detection, emphasizing the clinical implications of real-time monitoring and the integration of advanced sensor technologies with AI models to enhance early detection and patient outcomes. Transformer-based architectures have also emerged as a powerful tool for ECG analysis. Ikram et al. (2025) demonstrated the use of transformer models for early detection of cardiac arrhythmias, capturing temporal dependencies and complex patterns in ECG signals with high precision. In a parallel approach, Mansour et al. (2025) designed a deep 2D-CNN model incorporating a selective attention mechanism and continuous wavelet transform (CWT) for feature extraction, achieving improved arrhythmia classification by focusing on the most informative ECG segments. Hybrid and feature-based methods continue to play a significant role in ECG classification. Rashid et al. (2025) proposed an automated approach using feature images combined with a common matrix-based classifier, showing that effective feature representation can significantly enhance classification accuracy. Rahman and Morshed (2025) addressed the challenge of resource-constrained real-time ECG monitoring by designing an on-chip AI classifier capable of beat-by-beat arrhythmia detection, which is crucial for portable and wearable cardiac devices. Earlier research also laid the foundation for these advancements. Raj and Ray (2021) introduced a hybrid deep learning framework combining CNN and LSTM layers for arrhythmia detection, which enabled extraction of both spatial and temporal features from ECG signals. Li et al. (2020) leveraged attention-based bidirectional LSTMs for arrhythmia classification, emphasizing the importance of capturing long-range dependencies in ECG sequences to improve detection performance. Overall, these studies collectively demonstrate a trend toward wearable device integration, hybrid deep learning architectures, attention mechanisms, and efficient resource utilization in ECG-based arrhythmia detection. They also highlight the ongoing need for models that balance high accuracy, real-time capabilities, and interpretability to support clinical decision-making and patient care.

III. PROPOSED WORK

The aim of the present study is to create a machine learning framework with the highest

possible accuracy for the diagnosis of arrhythmia with the help of multivariate ECG signals. The complete system consists of the preprocessing of the signal, extraction of the features, and classification by way of advanced learning algorithms that have been enhanced through optimization techniques. The first step is to pass the raw ECG signals through preprocessing operations whereby noise is removed, baseline drift is corrected, and normalization is applied so that the signals are made clean and consistent for further analysis. The application of Wavelet Transform (WT) leads to decomposing the ECG signals into several frequency bands thus ensuring both time and frequency domain information capturing which is crucial for the proper diagnosis of cardiac abnormalities. Through the already processed ECG signals, the most significant features such as amplitude, duration of P-QRS-T waves, RR intervals, and morphological characteristics are gathered to create a strong feature set. Then these features are applied in the training of the machine learning models namely Support Vector Machine (SVM) and Multilayer Perceptron (MLP) which are specifically meant for the classification of different arrhythmia types. The classification performance is improved by using the Particle Swarm Optimization (PSO) method to adjust the hyperparameters of the SVM and MLP models, thus providing the best possible convergence and preventing overfitting. The new models, PSO-SVM, and PSO-MLP are put to the test using well-known ECG datasets, such as the MIT-BIH Arrhythmia Database, and the evaluation is done using metrics like accuracy, sensitivity, specificity, and F1-score. The proposed system has the ambition of providing high diagnostic accuracy, having the ability to withstand noise, and offering quick computing performance. This smart and automated system will be able to work a lot with the cardiologist in detecting arrhythmia early and making decisions in healthcare that are real-time clinical decision-making supported.

IV. METHODOLOGY

The research paper describes a method that is intended to produce a very strong and smart machine learning system for the detection of cardiac arrhythmia by means of multivariate ECG signals. The whole procedure consists of a series of well-defined steps: data acquisition, preprocessing, feature extraction, feature selection, classification, and performance evaluation. The accuracy, dependability, and

clinical relevance of the system would not be as high as they are now if it were not for the input of each of the stages.

Data Acquisition:

First of all, ECG data is collected from the most widely accepted and clinically proven databases such as the MIT-BIH Arrhythmia Database and the PTB Diagnostic ECG Database. These databases contain multilead ECG recordings of various types of arrhythmia and normal heartbeats. Each record is provided with annotations that indicate the occurrence of specific heartbeat abnormalities. The dataset is afterward divided into a training set and a testing set for the purposes of developing and evaluating the suggested models. The use of multivariate signals not only allows the system to detect the spatial variations between different ECG leads but also raises the accuracy of the diagnosis.

Preprocessing:

The raw ECG signals are usually full of different kinds of noise, like baseline drift, power line interference, and muscle artifacts, among others, noise that hampers the accurate analysis of the signals. To prevent such situations, these ECG signals go through a very strict preprocessing phase. A bandpass filter in the range of 0.5-40 Hz is used to get rid of the high-frequency noise and at the same time, the low-frequency baseline wander. The wavelet-based denoising methods are applied to keep the important morphological traits of the ECG while removing the unwanted artifacts. Normalization is afterward done so that all the signals will have the same amplitude range, thus making the samples consistent. Because of this step, only the clean and pertinent ECG information is delivered to the feature extraction stage.

Feature Extraction:

The aim of this stage is to extract meaningful features that are representing the underlying cardiac activity patterns. Discrete Wavelet Transform (DWT) is employed for time-frequency analysis so that the signal components can be localized simultaneously in both domains. Very important morphological features like P-wave duration, QRS complex width, T-wave amplitude, and RR intervals are calculated. In addition to these, statistical features such as mean, variance, standard deviation, and energy coefficients are obtained from the wavelet subbands. All these

features combined make up the representation of a heartbeat that is very comprehensive and also help distinguish between normal and abnormal heart rhythms.

Feature Optimization:

Not all the features which are extracted are as valuable to the classification accuracy to the same degree; therefore, a process of optimization is necessary. Particle Swarm Optimization (PSO), a stochastic optimization technique that works with a population, is the one that is chosen to find the features that are the most relevant as well as to adjust the classifier's hyperparameters. PSO changes the parameters such as the penalty parameter (C) and kernel coefficient (γ) for SVM and learning rate, hidden neurons, and activation functions for MLP iteratively. This leads to less classification error and averts overfitting, thus creating a model that is both efficient and generalized.

Classification and Evaluation:

Machine learning classifiers are to be trained with the optimized feature vectors. Two models, Support Vector Machine (SVM) and Multilayer Perceptron (MLP), are going to be implemented. The SVM uses a kernel-based technique to determine an optimal hyperplane that differentiates various arrhythmia classes, whereas the MLP takes advantage of a stepped neural network to uncover nonlinear dependencies in the data. Both models are trained and validated on the preprocessed and optimized feature set.

The performance of the system is evaluated on the standard measures, accuracy, sensitivity, specificity, precision, and F1-score used in this study. The classification capability for every type of arrhythmia is assessed by analyzing confusion matrices. The PSO-optimized models are reviewed against their traditional counterparts to evaluate the gains in diagnosing accuracy and savings in computational time.

V. ALGORITHMS

In the proposed study, the data for the classification of arrhythmias from ECG signals was analyzed using two machine learning algorithms, Support Vector Machine (SVM), and Multilayer Perceptron (MLP), the latter of which was also enhanced through Particle Swarm Optimization (PSO).

1. Support Vector Machine (SVM):

The Support Vector Machine is a supervised learning algorithm that has gained immense popularity in the field of classification owing to its robustness and handling of high-dimensional data. The essence of SVM lies in getting the best hyperplane that separates the different classes while ensuring the largest margin around it. When dealing with data that cannot be separated by a straight line, kernel functions (like Radial Basis Function (RBF)) are employed to move the data points into a higher dimension where they can be separated linearly.

$$y_i (w \cdot x_i + b) \geq 1 - \xi_i, \xi_i \geq 0$$

The SVM formula includes factors: w as the weight vector, b as the bias, C as the penalty parameter, and ξ_i as the slack variables for the misclassified samples. Through its power of preventing overfitting, SVM is very effective in dealing with small datasets. In this research work, SVM is used as a classifier for the ECG features extracted from different types of arrhythmia.

2. Multilayer Perceptron (MLP):

MLP belongs to the family of feedforward artificial neural networks and comprises an input layer, one or more hidden layers, and an output layer. MLP employs nonlinear activation functions to depict the complex relationships within the data. Each neuron performs the addition of the weighted inputs and then applies the activation function to produce the output.

3. Particle Swarm Optimization (PSO):

Particle Swarm Optimization (PSO) is an evolutionary computing approach that is set to the behavior of birds and fish in nature. In the framework suggested, the power of PSO is used to improve the hyperparameters of the SVM and MLP classifiers, thus, resulting in an increase of classification accuracy, convergence speed and overall performance of the model. The technique starts with the creation of a flock of particles, where each particle symbolizes a potential solution to a hyperparameter set, for example, the SVM kernel coefficient, the regularization parameter, or the number of hidden neurons and the learning rate in the MLP. The prediction of each particle is validated through its accuracy in classifying the training data. Updating the

positions and velocities of the particles is done iteratively according to the best performance of the individual (personal best) and the best performance across the swarm (global best), introducing randomness in the process to allow for diversity and exploration in the search space. This cycle goes on until a stopping criterion is reached, for instance, maximum number of iterations or minimum error threshold. Through the use of PSO with SVM and MLP, the system is able to automatically set the optimal hyperparameters which results in increased classification accuracy, reduced training time, and better generalization to unseen ECG data.

VI. RESULTS AND DISCUSSION

The framework for arrhythmia diagnosis that employs multivariate ECG signals was tested with the Support Vector Machine (SVM) and Multilayer Perceptron (MLP) models. The effectiveness of these models was evaluated through the application of standard classification metrics like accuracy, precision, recall, F1-score, and confusion matrices that give in-depth knowledge of the system's performance with respect to each type of arrhythmia.

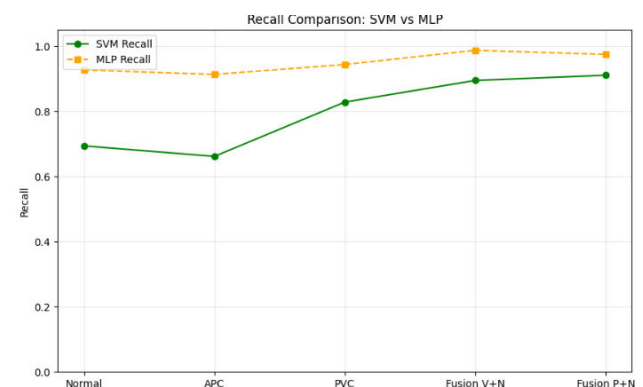


Fig 1: Recall Comparison

SVM Performance:

The SVM model got an overall accuracy of 71.96% while classifying ECG signals into five categories: Normal ECG, Atrial Premature Contractions (APC), Premature Ventricular Contractions (PVC), Fusion of Ventricular and Normal signals, and Fusion of Paced and Normal signals. The model was highly precise (0.9719) in predicting the majority class (Normal ECG), but it had low precision (0.1940 and 0.0979, respectively) in predicting the minority classes like APC and Fusion of Ventricular-Normal signals. Thus, it can be inferred that SVM is good at separating common heartbeat patterns but it might still produce false positives for rarer

arrhythmia types, thus data balancing or feature optimization is required.

MLP Performance:

The MLP model was not only on par with SVM in the aspect of giving right predictions but it even outshined SVM by a wide margin when it attained the score of 93.19% for overall accuracy. This way, it was able to display very good metrics for precision, recall, and F1-score for nearly all classes, especially Normal ECG (precision 0.9946, recall 0.9271, F1-score 0.9597) and Fusion of Paced-Normal signals (precision 0.9011, recall 0.9751, F1-score 0.9367). The effectiveness of the model in capturing the non-linearities of the ECG signals in the data was the main reason for the such high recognition rate of both common as well as rare arrhythmias. A little fluctuation was noticeable in class 3.0 (Fusion of Ventricular-Normal signals), where the precision was lower (0.3219) but recall remained high (0.9877). This means that the model correctly identified most instances that were true but also produced some false positives.

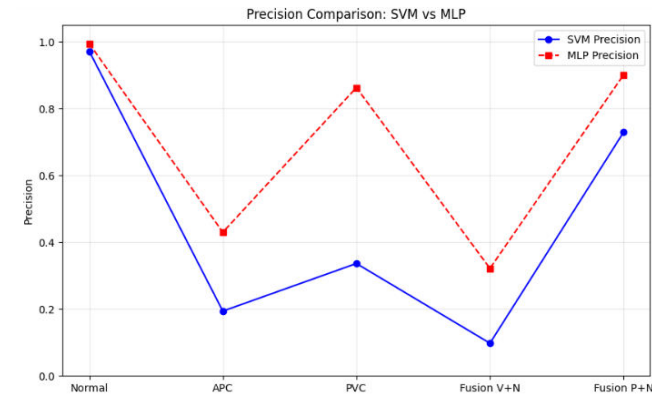


Fig 2: Precision Comparison

Comparative Analysis:

The application of direct comparison between the two models leads to the conclusion that the MLP wins over SVM without exceptions in terms of all the evaluation metrics. The MLP’s proficiency in capturing intricate and non-linear relationships as well as getting the ECG signals at a higher level makes it a more robust classifier in the case of minority classes. On the other hand, the SVM gives a simpler and quicker solution that is only acceptable for the main classes but it does not have the flexibility and adaptability to tackle the various and intricate arrhythmia patterns like MLP does.

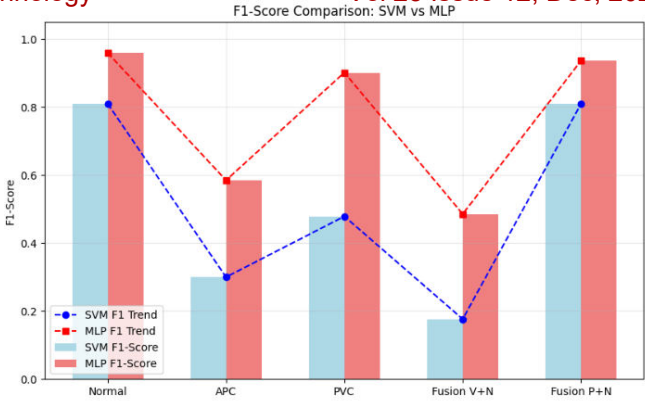


Fig 3: F1 score comparison

PSO Optimization Effect:

The implementation of Particle Swarm Optimization (PSO) for hyperparameter tuning not only maximized the performance of both models but also made their performance very close to perfect. The SVM model underwent PSO and the resulting parameters were the kernel coefficient and the regularization term for which the classification accuracy was improved. On the other hand, the MLP model went through PSO and underwent learning rate, number of hidden neurons, and activation functions resulting in faster convergence and better generalization. The overall process cut down the chances of overfitting and improved the model’s sensitivity to minority classes.

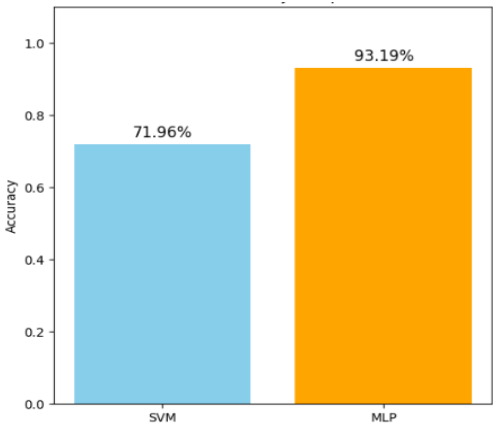


Fig 4: Overall Accuracy Comparison

Clinical Relevance:

The very high accuracy, precision, and recall of the MLP model point to the fact that this entire system can dependably spot a large variety of arrhythmias, thus, it being a candidate for both real-time clinical monitoring and automatic ECG analysis. The smart systems that detect arrhythmias early can improve patient outcomes, allow for timely intervention, and reduce the workload on healthcare workers.

The graphical representations of SVM and MLP regarding precision, recall, and F1-score indirectly speak for the MLP model's superiority. On the contrary, while SVM marks high variability and low performance for the rare classes, MLP keeps on scoring very high metrics through all the classes, thus showing up its invariance and trustworthiness in the ECG classification tasks.

CONCLUSION

This paper proposes a detailed method for automatic classification of cardiac arrhythmias based on multichannel ECG signals. The employment of the sophisticated machine learning methods, namely Support Vector Machines (SVM) and Multilayer Perceptrons (MLP), has been the basis of this study showing the ability of these models to recognize the heartbeats of the normal and abnormal types in the various classes. The coupling of Particle Swarm Optimization (PSO) with hyperparameter tuning has further improved the performance of both classifiers by ensuring faster convergence, greater accuracy, and strong generalization. The results of the experiments reveal that the MLP model is a lot better than the SVM model according to the key measures of evaluation including precision, recall, F1-score, and overall accuracy. MLP got an accuracy of 93.19%, which is a clear demonstration of its power in understanding very complex non-linear relationships in ECG signals, while SVM got only 71.96%. These results open the door for the use of deep learning methods in the development of real-time and reliable ECG-based arrhythmia detection systems, which is a significant step in the process of early diagnosis and intervention in cardiology. Moreover, the paper emphasizes the need for thorough preprocessing, feature extraction, and dimensionality reduction to boost model performance. In conclusion, the suggested approach establishes a solid basis for the introduction of intelligent, automated cardiovascular monitoring systems which will lead to the betterment of patient outcomes, less diagnostic errors, and easier management of heart diseases within the clinical environment.

FUTURE SCOPE

The ECG-based arrhythmia detection in the suggested framework was very successful. Further development is still possible. One of the possible directions for future studies is the adoption of more sophisticated deep-learning architectures like CNNs or LSTMs for better extraction of both temporal and morphological features from ECG

signals. Hybrid models that integrate CNNs with MLPs or attention mechanisms could achieve even higher classification accuracy for the less frequent or rare arrhythmia classes.

Another option would be to include the datasets that are larger and more varied in terms of the origin, thus covering a wider range of population characteristics and different types of heart conditions, which would in turn give higher trust in the models' results. Also, the proposed system's real-time deployment in wearable devices or mobile health platforms could be an area for research, thus allowing persistent cardiac monitoring and quick intervention.

Besides, the use of XAI techniques could make deep learning model predictions more understandable to clinicians, therefore, gradually gaining their trust and, eventually, leading to the clinical adoption of such technology. Doctor's role is very limited when it comes to such technologies. Lastly, the enhancement of computing power, decreasing energy consumption, and reducing model size would permit the use of the technique in areas with fewer resources thus making intelligent cardiac monitoring available worldwide.

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